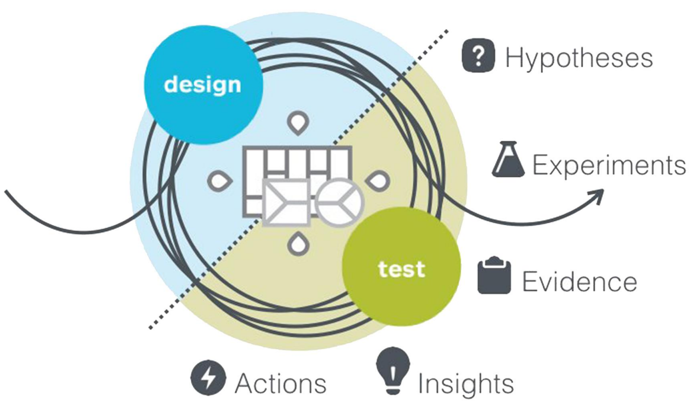


3 Days Online Workshop on

Design of Experiments- An Engineering Perspective

22, 23 & 24th April 2020

jaeronline.com/doeworkshop

ORGANISED BY

JAER

HEXACUBE

RESPONSE SURFACE METHODOLOGY (RSM)

INTRODUCTION

1. Response surface methodology is a specialized DOE technique.
2. RSM is a combination of statistical and optimization methods, used to model and optimize designs.
3. RSM comprises a group of statistical techniques for empirical model building and model exploitation.
4. By careful design and analysis of experiments, it seeks to relate a response, or output variable, to the levels of a number of predictors, or input variables, that affect it.

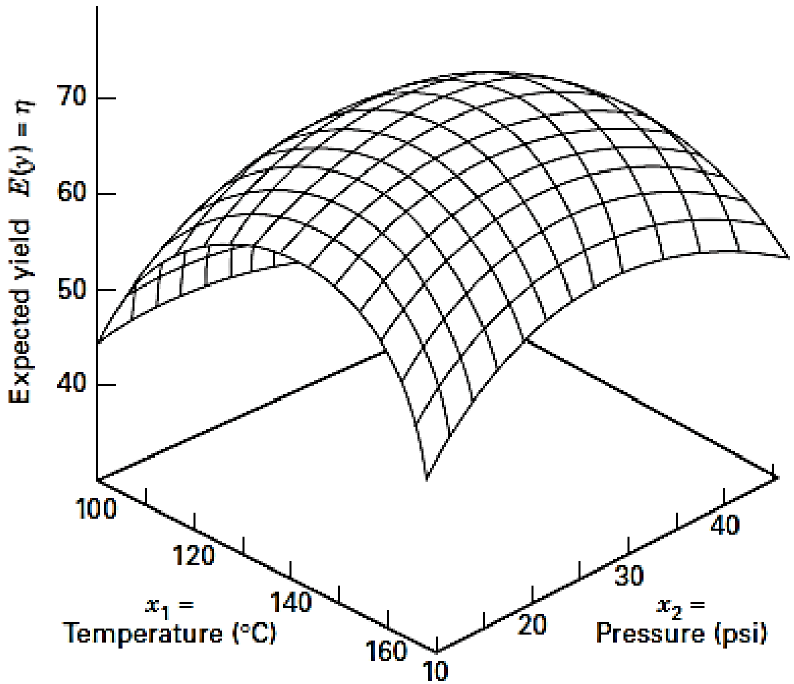
INTRODUCTION

1. A chemical engineer wishes to find the levels of temperature (x_1) and pressure (x_2) that maximize the yield (y) of a process.
2. The process yield is a function of the levels of temperature and pressure, say $$y = f(x_1, x_2) + \epsilon$$

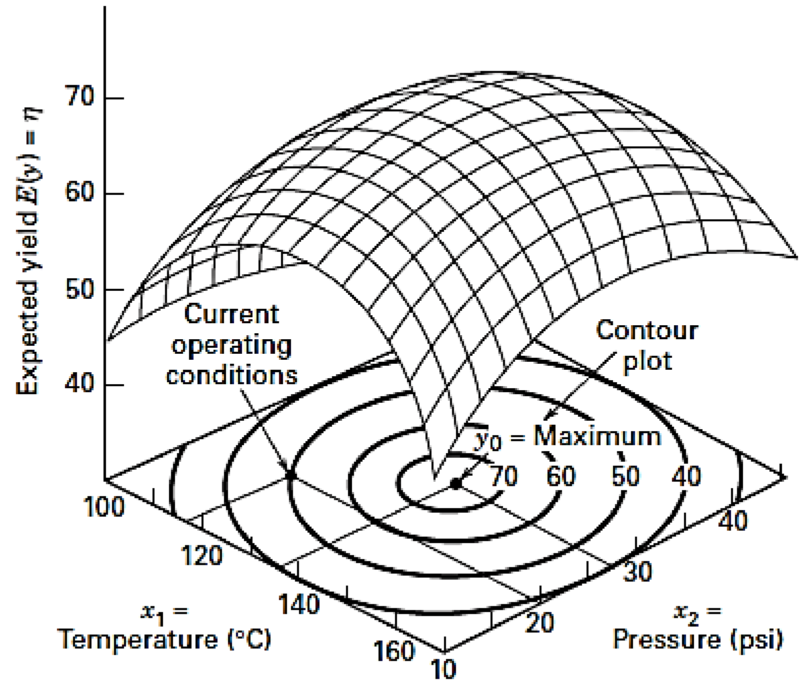
where ϵ represents the noise or error observed in the response y.

3. If we denote the expected response by $E(y) = f(x_1, x_2) = \eta$, then the surface represented by $\eta = f(x_1, x_2)$ called as Response Surface.

INTRODUCTION



■ **FIGURE 11.1** A three-dimensional response surface showing the expected yield (η) as a function of temperature (x_1) and pressure (x_2)



■ **FIGURE 11.2** A contour plot of a response surface

INTRODUCTION

1. First step in RSM is to find a suitable approximation for the true functional relationship between y and the set of independent variables.
2. Usually, a low-order polynomial in some region of the independent variables is employed.
3. If the response is well modeled by a linear function of the independent variables, then the approximating function is the first-order model. $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k + \epsilon$

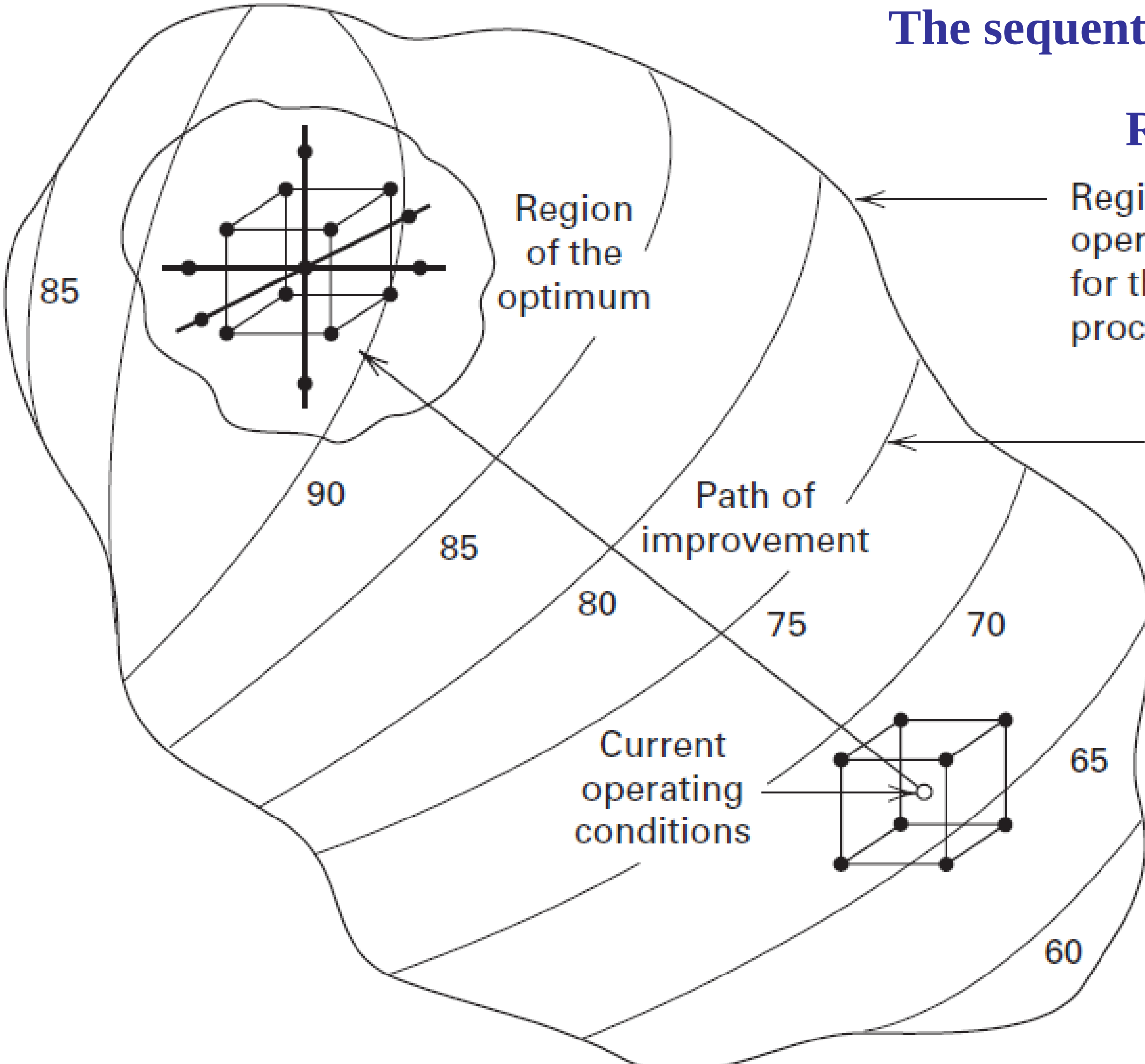
INTRODUCTION

1. If there is curvature in the system, then a polynomial of higher degree must be used, such as the second-order model.

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i<j}\sum \beta_{ij} x_i x_j + \epsilon$$

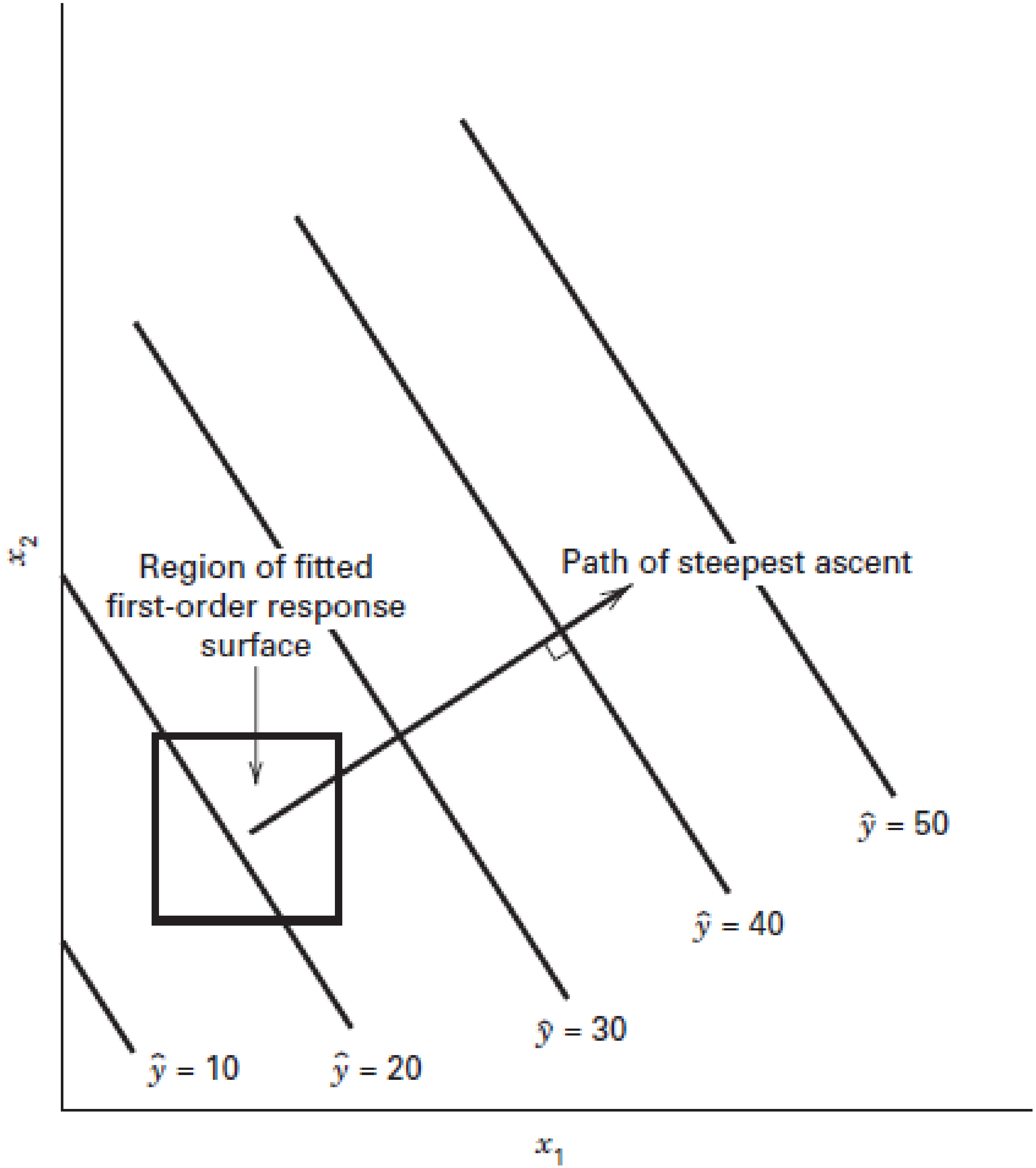
2. The method of least squares, is used to estimate the parameters in the approximating polynomials.
3. The response surface analysis is then performed using the fitted surface.
4. Designs for fitting response surfaces are called response surface designs.

The sequential nature of RSM



INTRODUCTION

1. The initial estimate of the optimum operating conditions for the system will be far from the actual optimum.
2. In such circumstances, the objective of the experimenter is to move rapidly to the general vicinity of the optimum.
3. The method of steepest ascent is a procedure for moving sequentially in the direction of the maximum increase in the response.
4. Of course, if minimization is desired, then we call this technique the method of steepest descent.

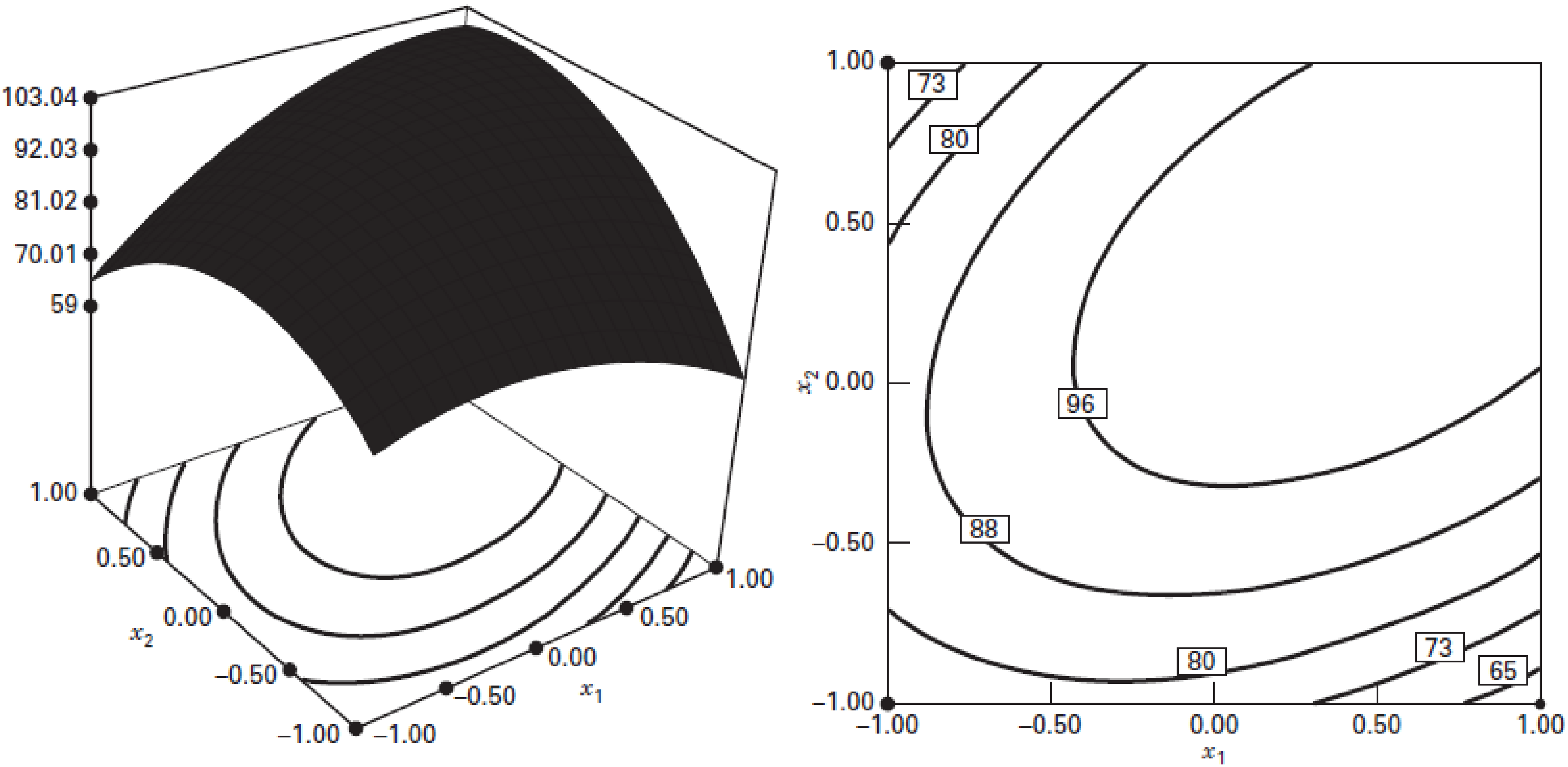


x_2
Region of fitted first-order response surface
Path of steepest ascent
$\hat{y} = 50$
$\hat{y} = 40$
$\hat{y} = 30$
$\hat{y} = 10$
$\hat{y} = 20$
x_1

STATIONARY POINT

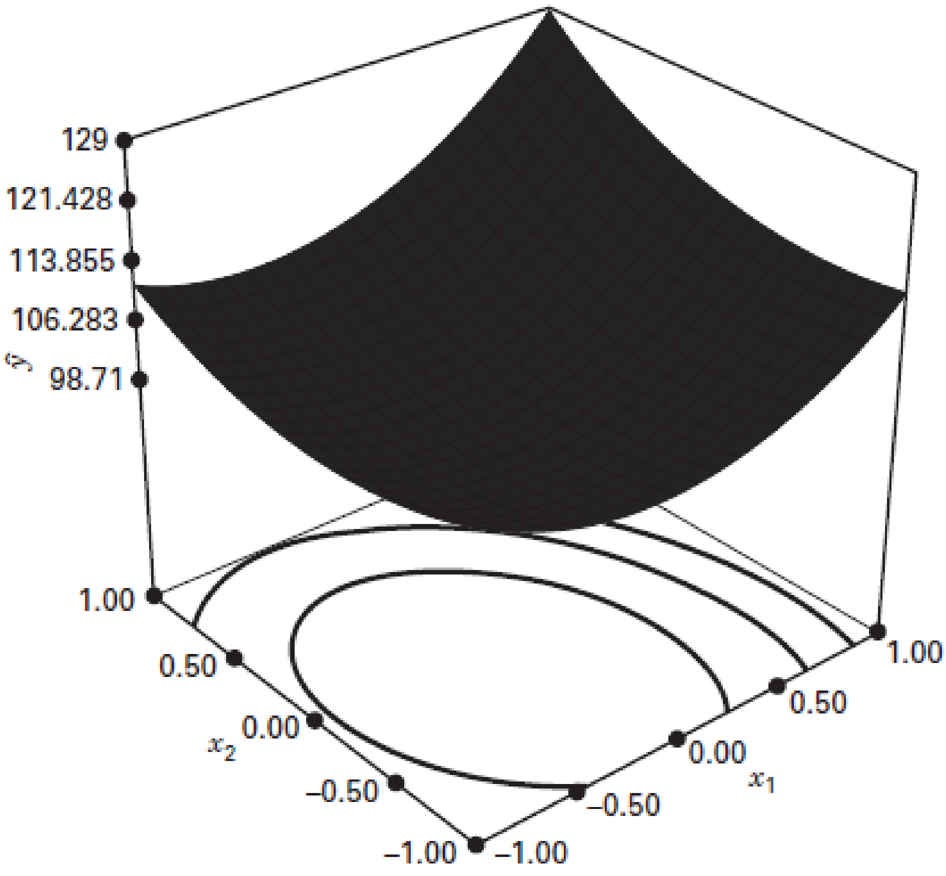
1. Find the levels of $x_1, x_2, \ldots, x_k$ that optimize the predicted response.
2. This point, if it exists, will be the set of $x_1, x_2, \ldots, x_k$ for which the partial derivatives $\partial\hat{y}/\partial x_1 = \partial\hat{y}/\partial x_2 = \cdots = \partial\hat{y}/\partial x_k = 0$.
3. This point, say $x_{1,s}, x_{2,s}, \ldots, x_{k,s}$, is called the stationary point.
4. The stationary point could represent a point of maximum response, a point of minimum response, or a saddle point.

SURFACE WITH MAXIMUM

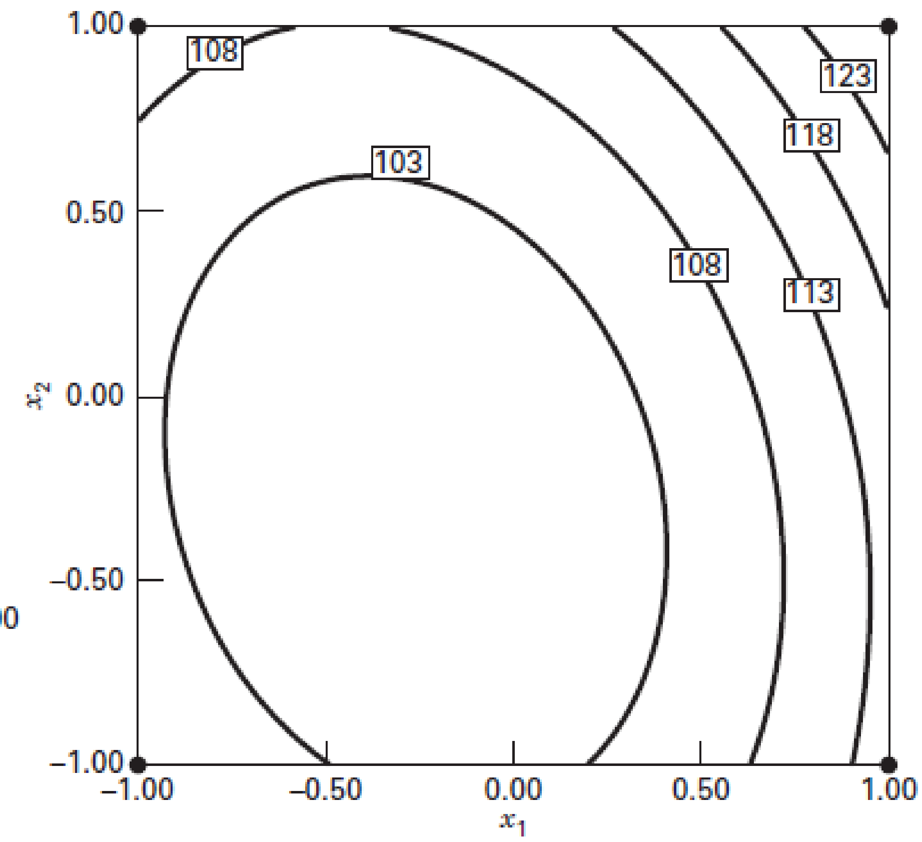


(*a*) Response surface (*b*) Contour plot

SURFACE WITH MINIMUM

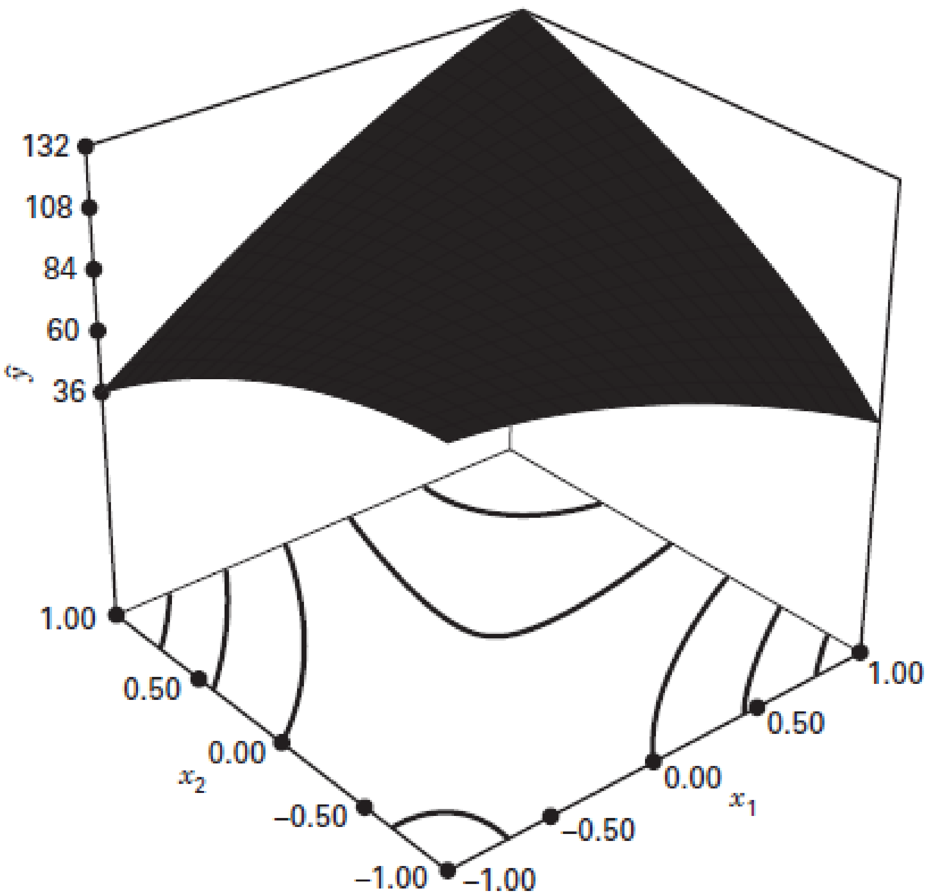


(a) Response surface

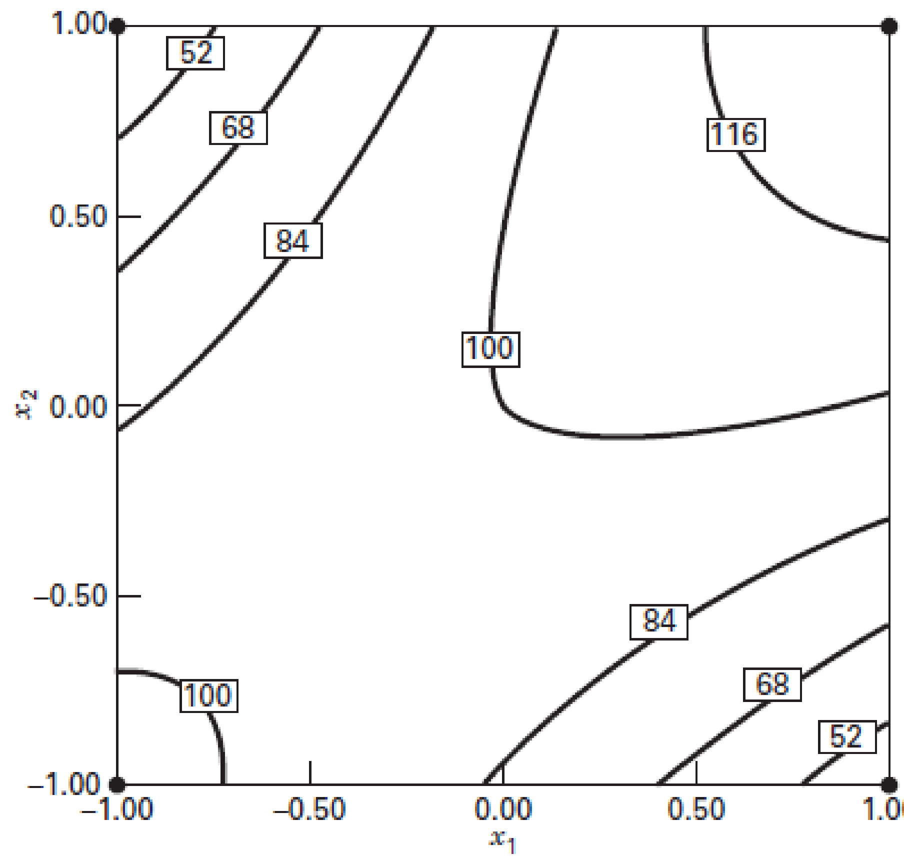


(b) Contour plot

SURFACE WITH SADDLE POINT (MINIMAX)

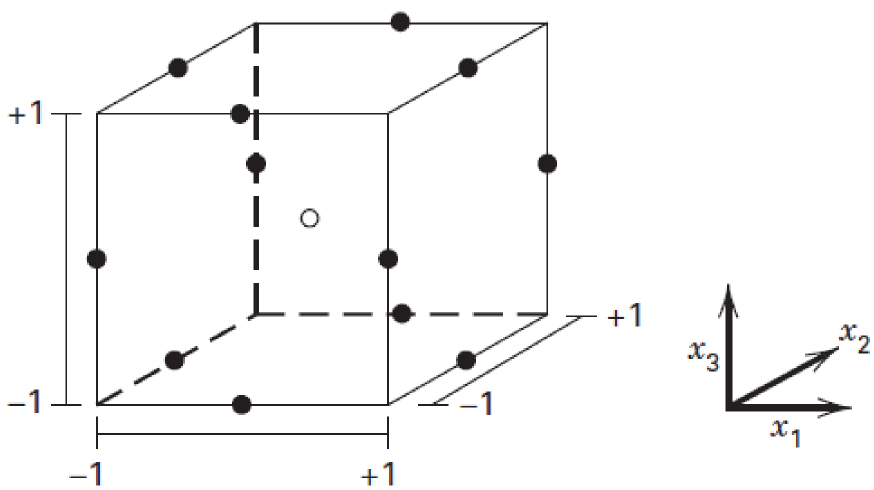


(a) Response surface

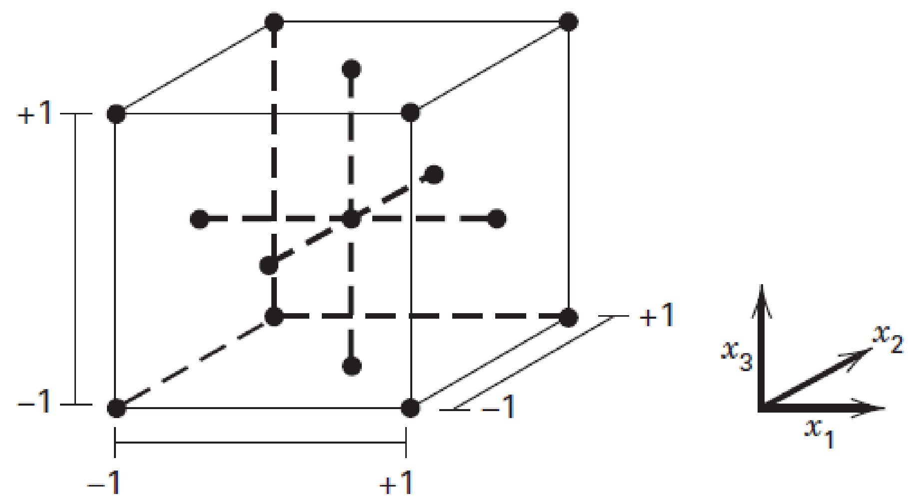


(b) Contour plot

RSM EXPERIMENTAL DESIGNS

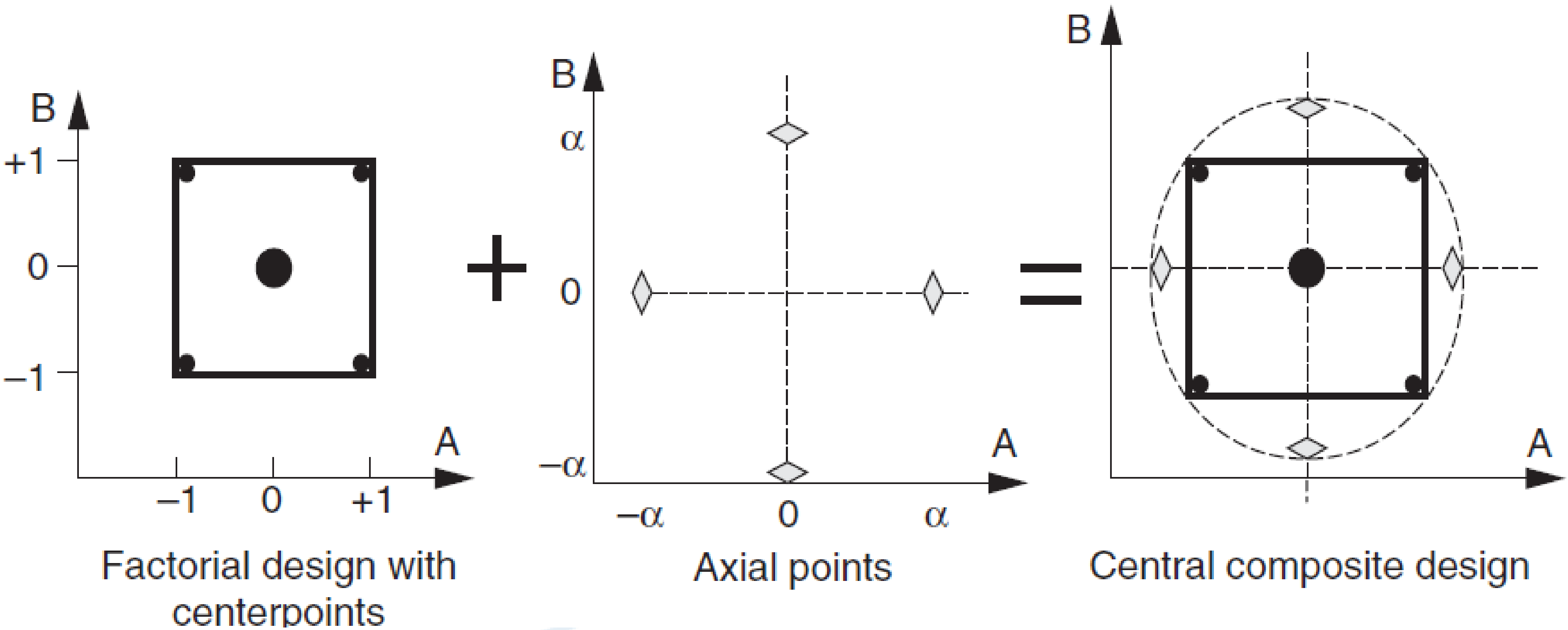


■ FIGURE 11.22 A Box–Behnken design for three factors



■ FIGURE 11.23 A face-centered central composite design for $k = 3$

RSM EXPERIMENTAL DESIGNS



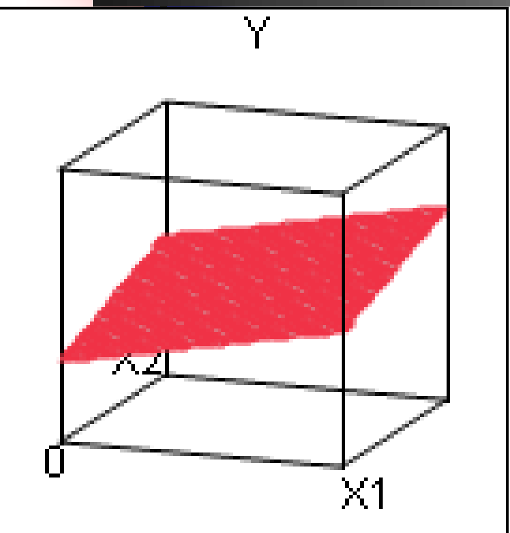
RSM EXPERIMENTAL DESIGNS

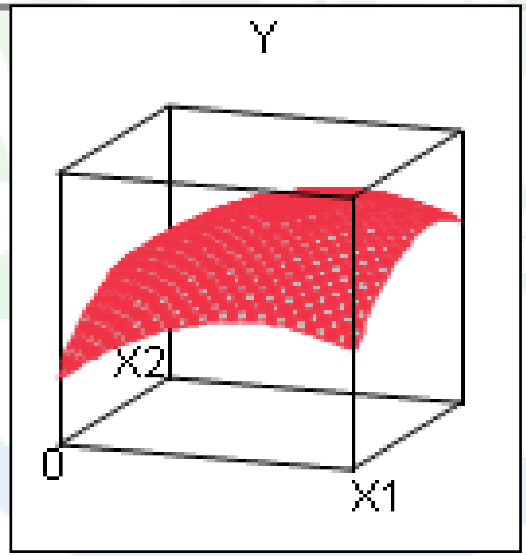
1. Rotatability is a reasonable basis for the selection of a response surface design.
2. Because the purpose of RSM is optimization and the location of the optimum is unknown prior to running the experiment, it makes sense to use a design that provides equal precision of estimation in all directions.
3. A central composite design is made rotatable by the choice of α.
4. The value of α for rotatability depends on the number of points in the factorial portion of the design.

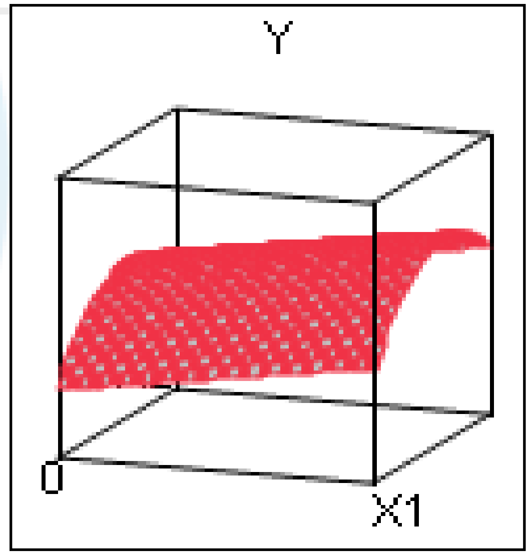
RSM EXPERIMENTAL DESIGNS

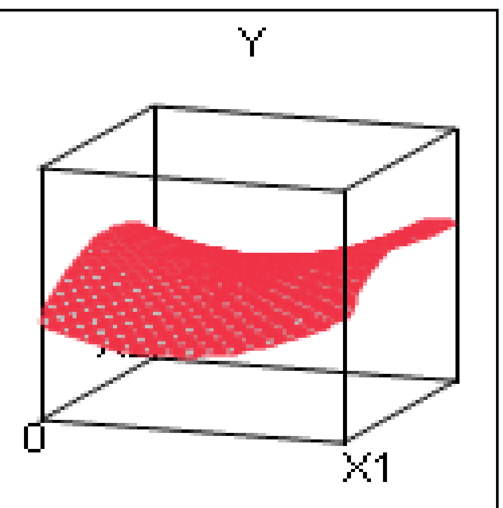
1. In fact, $\alpha = (n_F)^{1/4}$ yields a rotatable central composite design where n_F is the number of points used in the factorial portion of the design.
2. The Spherical CCD. Rotatability is a spherical property; that is, it makes the most sense as a design criterion when the region of interest is a sphere.
3. Center Runs in the CCD. The choice of in the CCD is dictated primarily by the region of interest. Generally, three to five center runs are recommended.
4. The Box–Behnken Design.

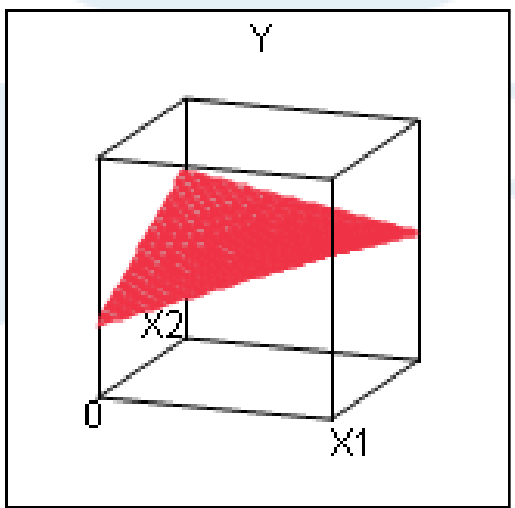
TYPES OF 3D SURFACES IN RSM











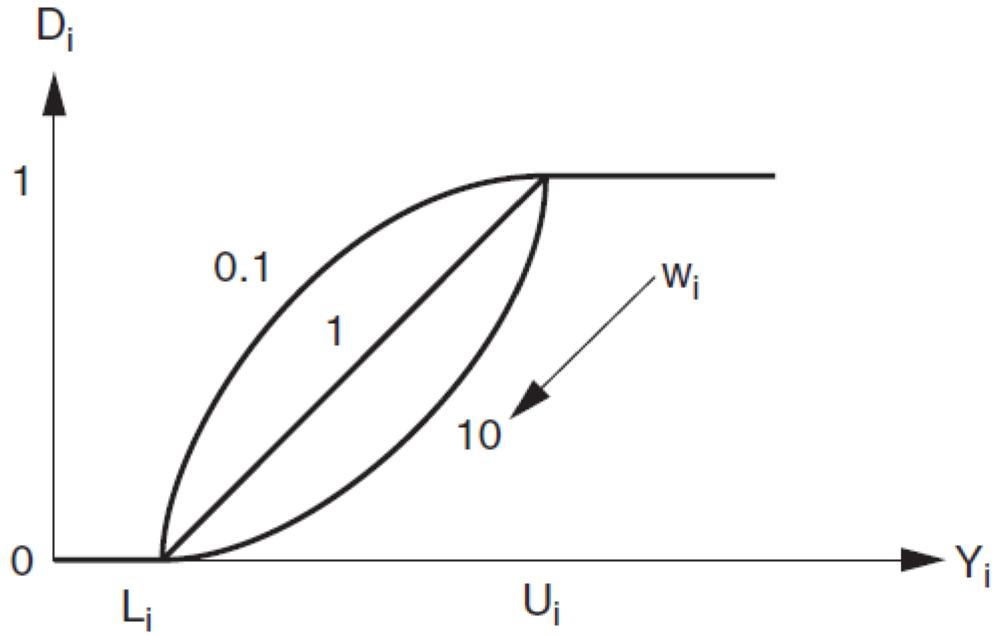
1. Planar Surface
2. Quadratic Surface
3. Ridge Surface
4. Saddle Surface
5. Twisted Surface

DESIRABILITY FUNCTION

1. The desirability function approach is one of the most popular methods used in the optimization of multiple-response surfaces.
2. Clearly, higher overall desirability D should indicate a higher overall satisfaction for all responses (requirements).
3. During the desirability approach, desirability function assigns values in the range of 0 and 1 to the probable values, with 0 for the "most undesirable values" and 1 for the "most desirable values".
4. The upper range of desirability value overall should specify a greater total fulfilment for all outputs.

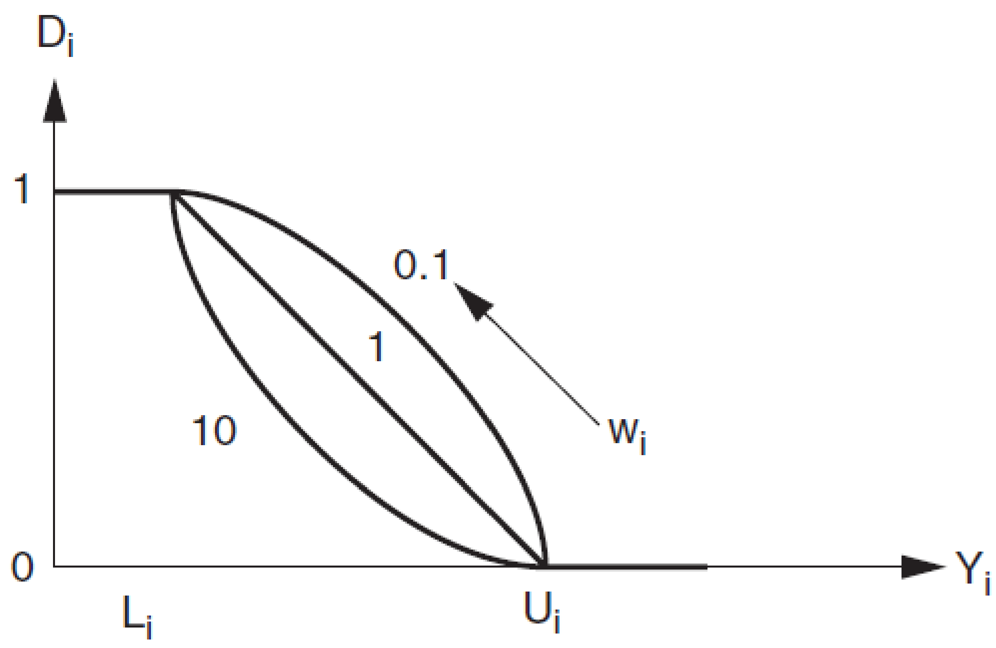
DESIRABILITY - Larger the Better

1. In this case a maximization of y_i is the most desirable result



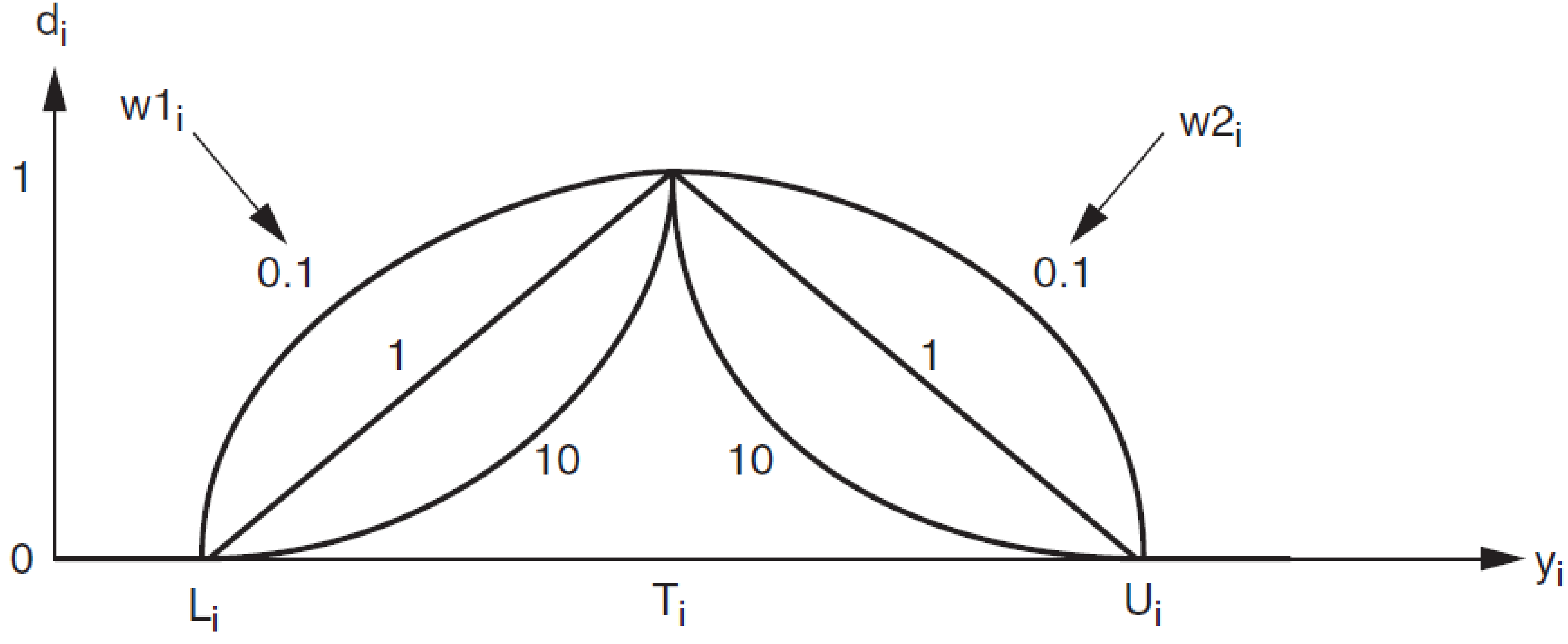
DESIRABILITY - Lower the Better

1. In this case a minimization of y_i is the most desirable result



DESIRABILITY - Nominal the Better

1. In this case, there is a target value T_i for y_i; the most desirable result is $y_i = T_i$.



DESIRABILITY

1. In In the equations, *A* and *B* are limit values and the exponent *s* determines the importance of achieving the target value; *X* is the input vector and f_r is the model used in the prediction.
2. *R* – no. of desirability function *(RESPONSES)*

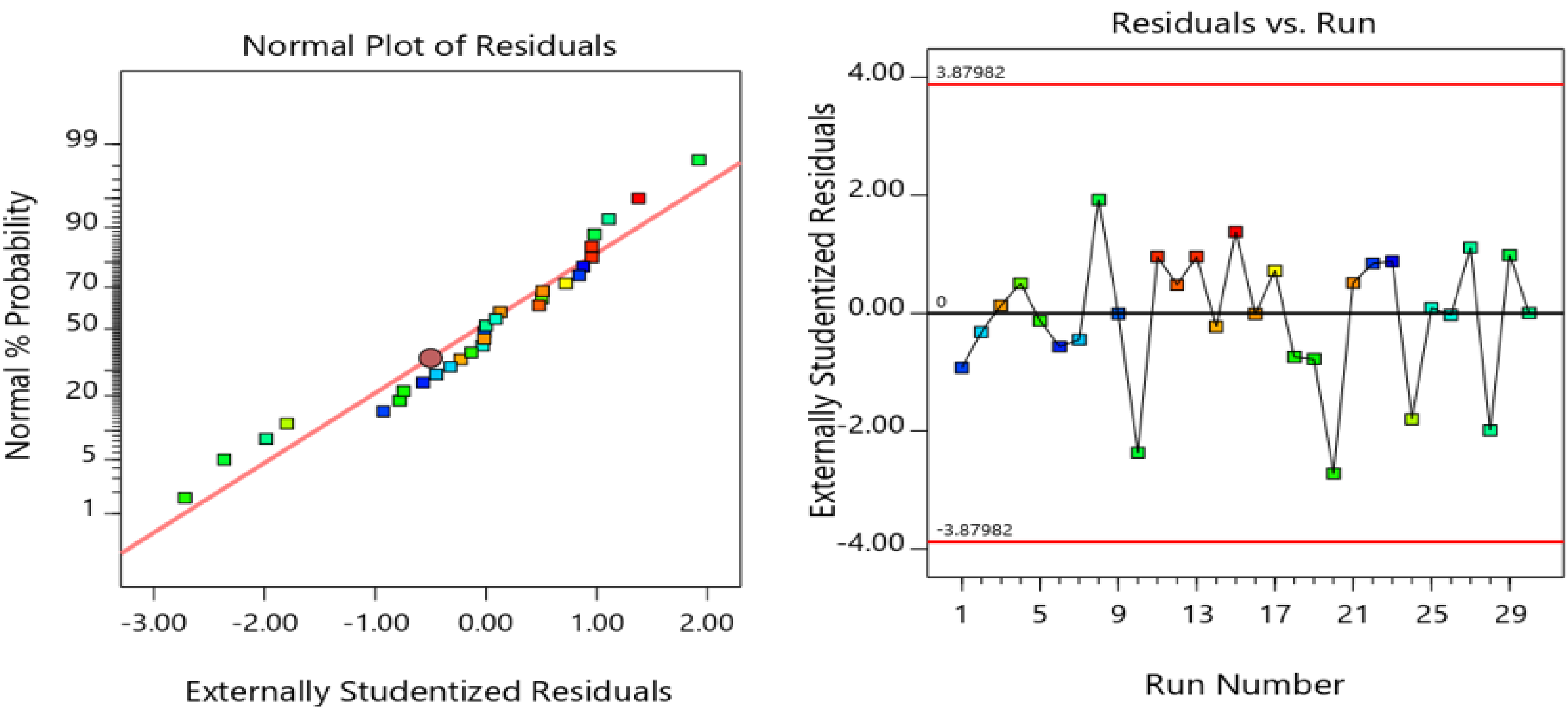
$$d_r^{max} = \begin{cases} 0 & \text{if } f_r(X) < A \\ \left(\dfrac{f_r(X) - A}{B - A}\right)^S & \text{if } A \leq f_r(X) \leq B \\ 1 & \text{if } f_r(X) > B \end{cases}$$

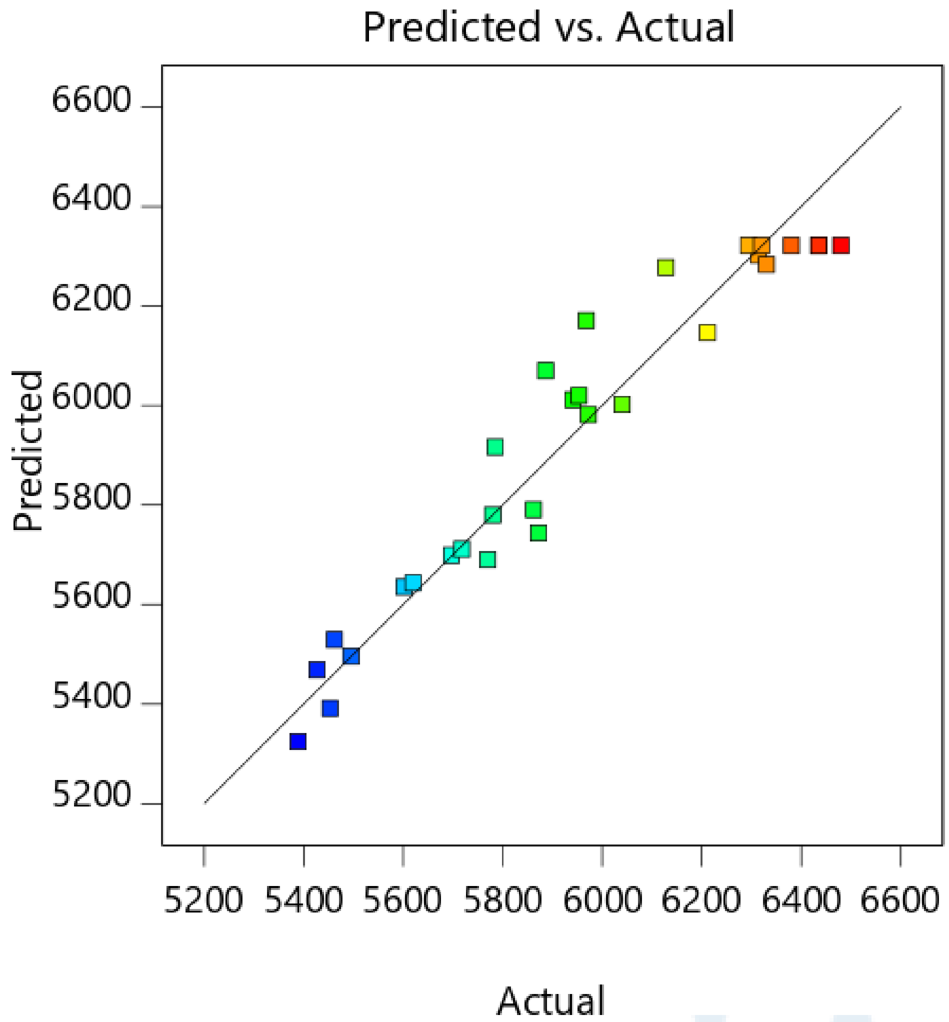
$$d_r^{min} = \begin{cases} 1 & \text{if } f_r(X) < A \\ \left(\dfrac{f_r(X) - B}{A - B}\right)^S & \text{if } A \leq f_r(X) \leq B \\ 0 & \text{if } f_r(X) > B \end{cases}$$

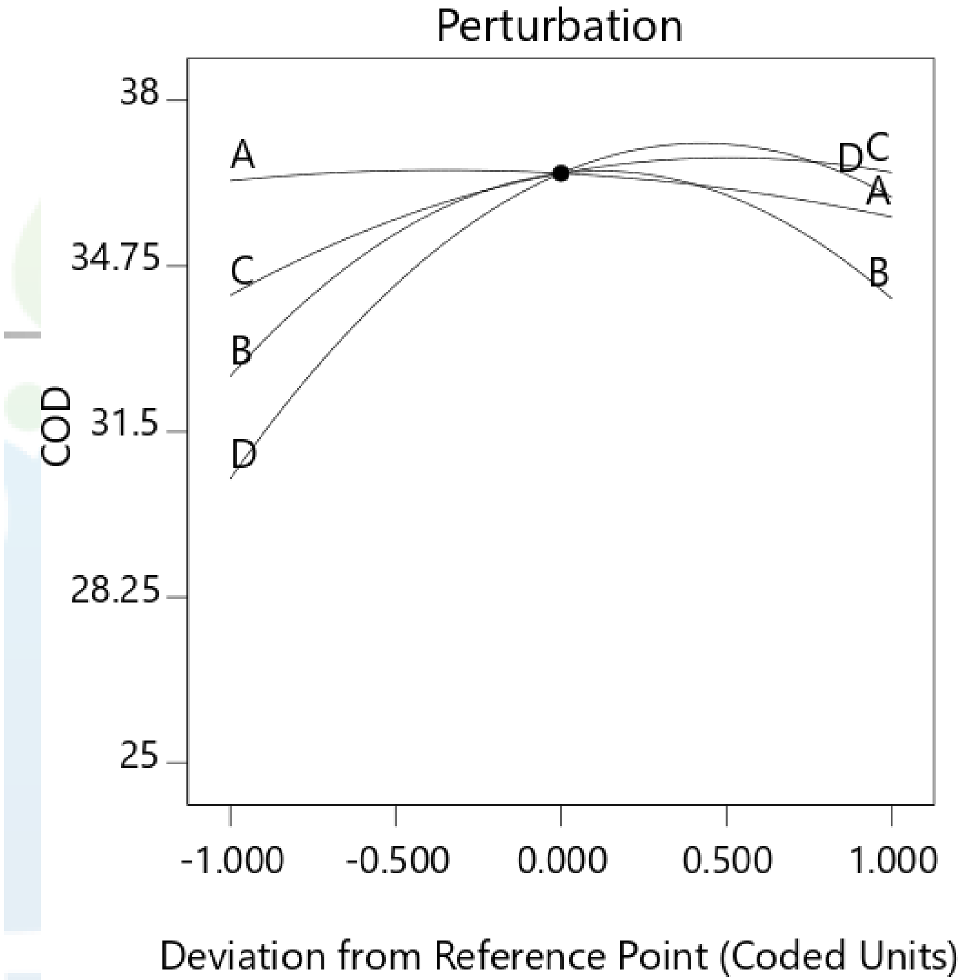
$$D = \left(\prod_{r=1}^{R} d_r\right)^{1/R}$$

VARIOUS PLOTS IN RSM

1. Normal Probability plot - The normal probability plot indicates whether the residuals follow a normal distribution, thus follow the straight line.
2. Residual vs. Run plot - It checks for lurking variables that may have influenced the response during the experiment. The plot should show a random scatter. Blocking and randomization provide insurance against trends ruining the analysis.
3. Predicted vs. Actual plot - The purpose is to detect a value, or group of values, that are not easily predicted by the model.

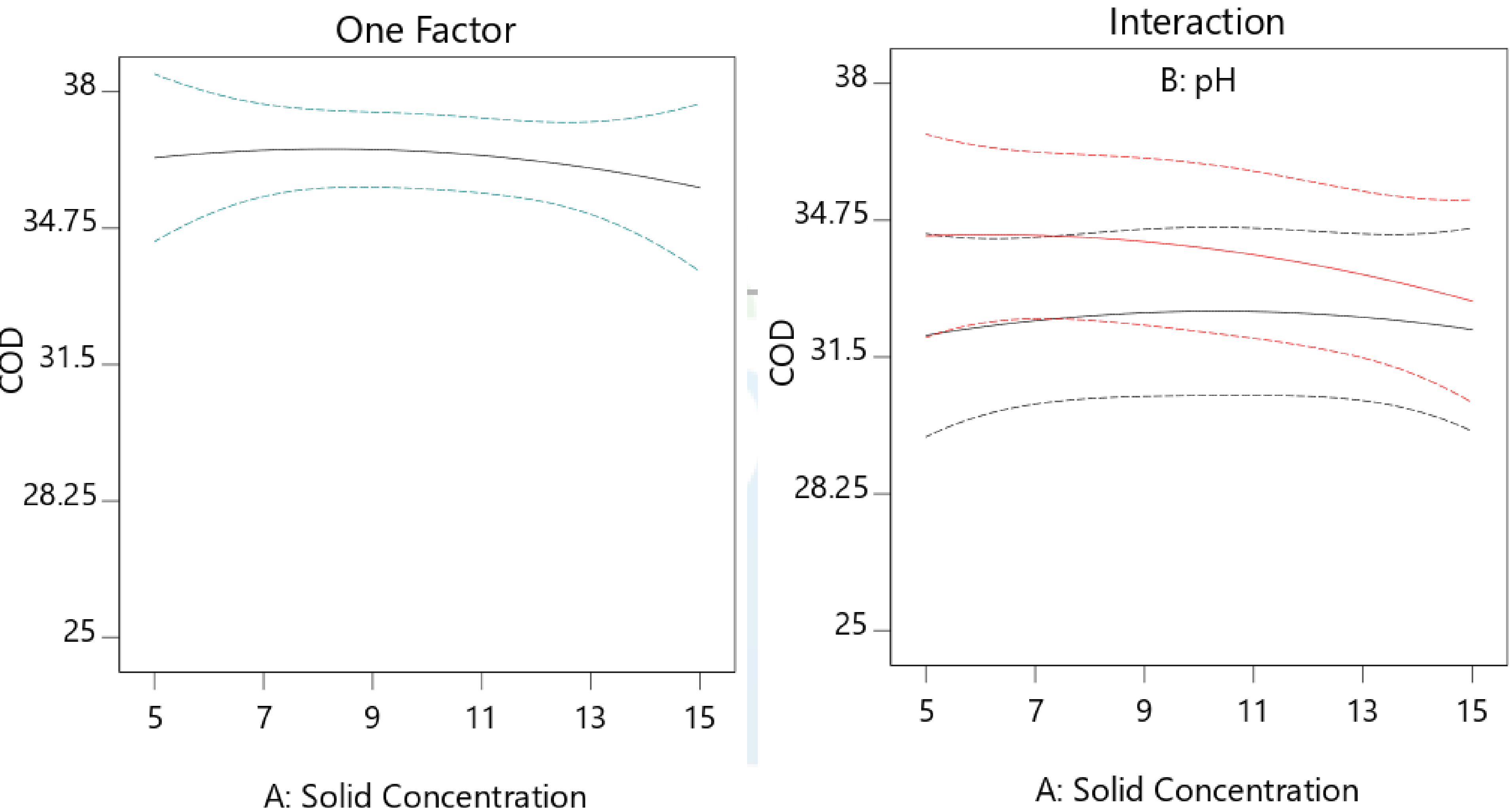

Normal Plot of Residuals
Normal % Probability
99
90
70
50
20
5
1
-3.00
-2.00
-1.00
0.00
1.00
2.00
Externally Studentized Residuals
Residuals vs. Run
Externally Studentized Residuals
4.00
2.00
0.00
-2.00
-4.00
3.87982
0
-3.87982
1
5
9
13
17
21
25
29
Run Number


Predicted vs. Actual
6600
6400
6200
6000
5800
5600
5400
5200
Predicted
5200 5400 5600 5800 6000 6200 6400 6600
Actual


Perturbation
38
34.75
31.5
28.25
25
COD
A
C
B
D
-1.000 -0.500 0.000 0.500 1.000
Deviation from Reference Point (Coded Units)

VARIOUS PLOTS IN RSM

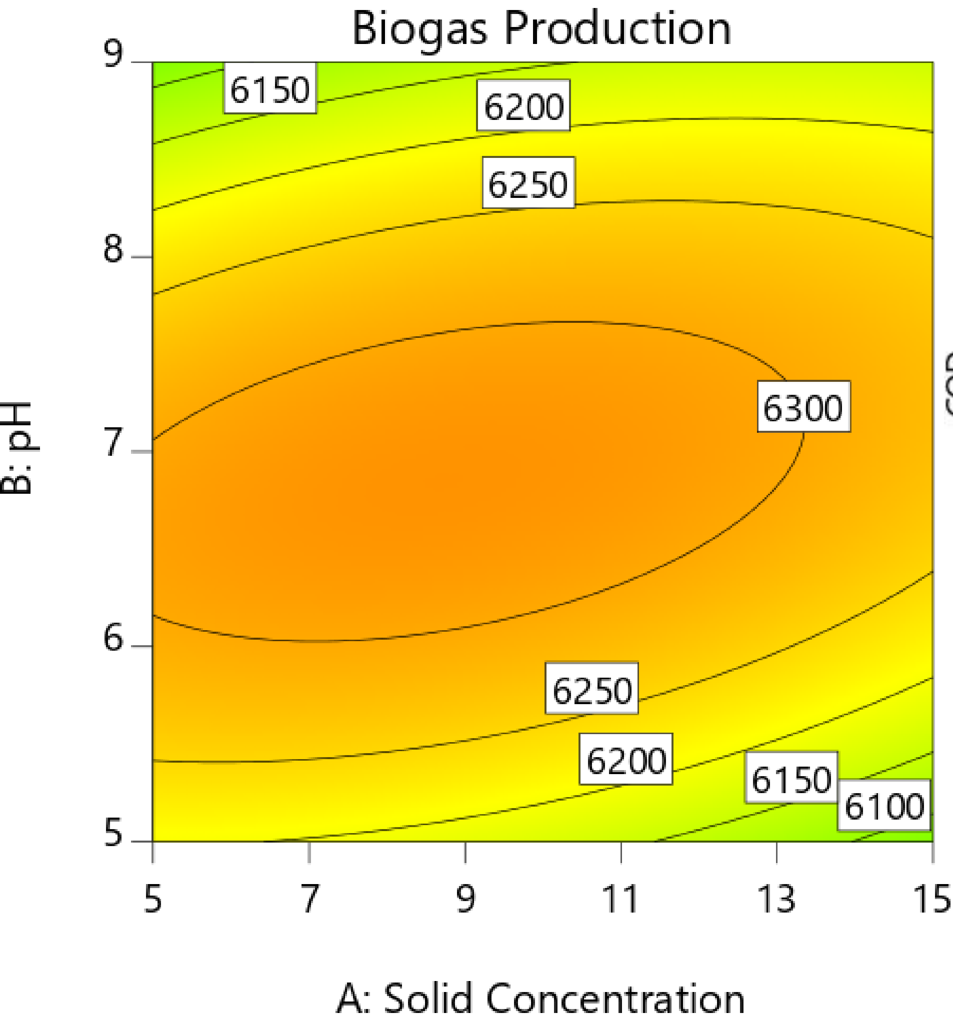
4. Perturbation plot - The perturbation plot helps to compare the effects of all the factors at a particular point in the design space. The response is plotted by changing only one factor over its range while holding all the other factors constant.
5. One Factor plot - The One Factor Effects graph shows the linear effect of changing the level of a single factor. It is constructed by predicting the responses for the low (-1) and high (+1) levels of a factor.

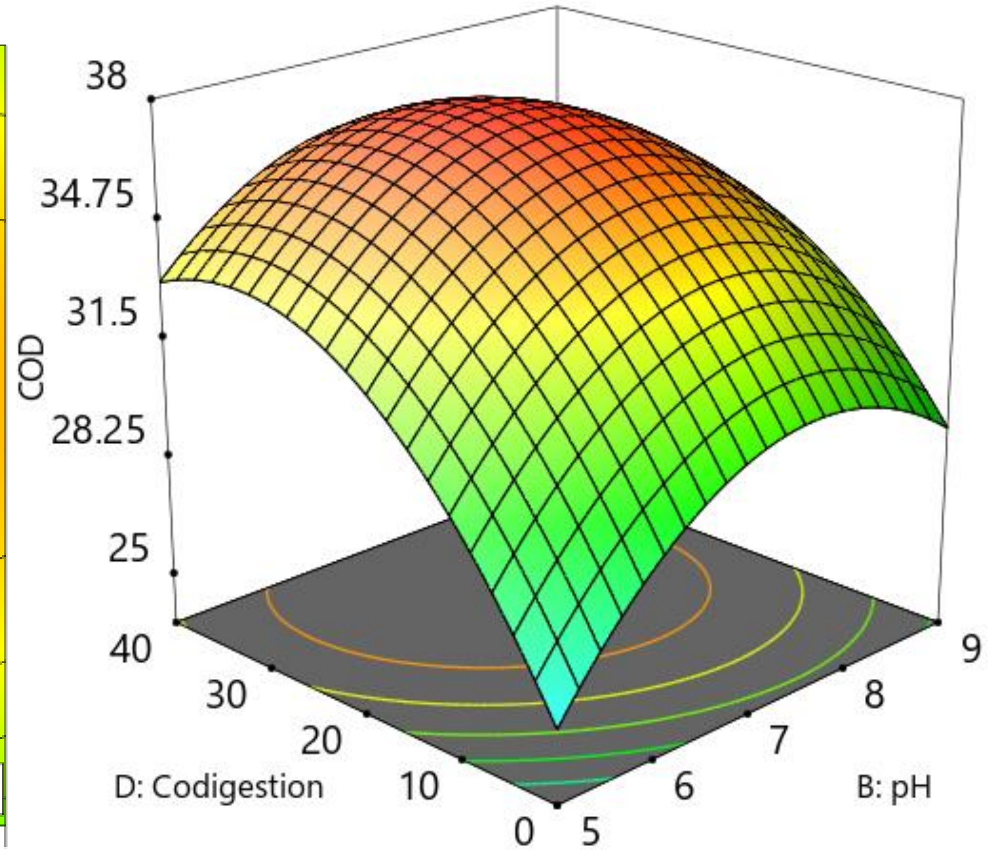


One Factor
38
34.75
31.5
28.25
25
COD
5
7
9
11
13
15
A: Solid Concentration
Interaction
B: pH
38
34.75
31.5
28.25
25
COD
5
7
9
11
13
15
A: Solid Concentration

VARIOUS PLOTS IN RSM

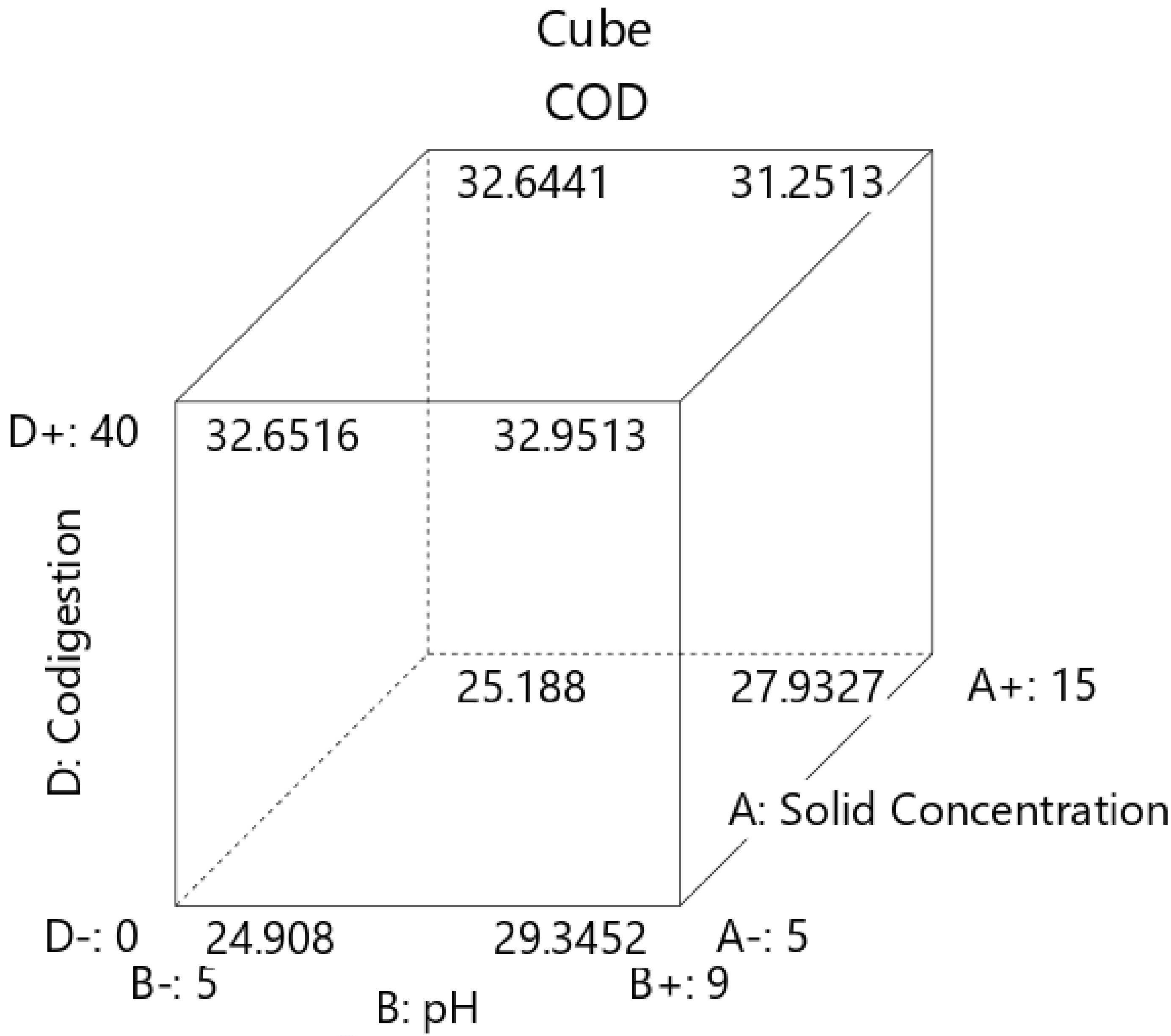
6. Interaction plot - An interaction occurs when the response is different depending on the settings of two factors. They will appear with two non-parallel lines, indicating that the effect of one factor depends on the level of the other.
7. Contour plot - The contour plot is a two-dimensional (2D) representation of the response plotted against combinations of factors, which shows the relationship between the response and factors.


Biogas Production
9
8
7
6
5
B: pH
6150
6200
6250
6300
6250
6200
6150
6100
5
7
9
11
13
15
A: Solid Concentration


38
34.75
31.5
28.25
25
COD
40
30
20
10
0
D: Codigestion
5
6
7
8
9
B: pH

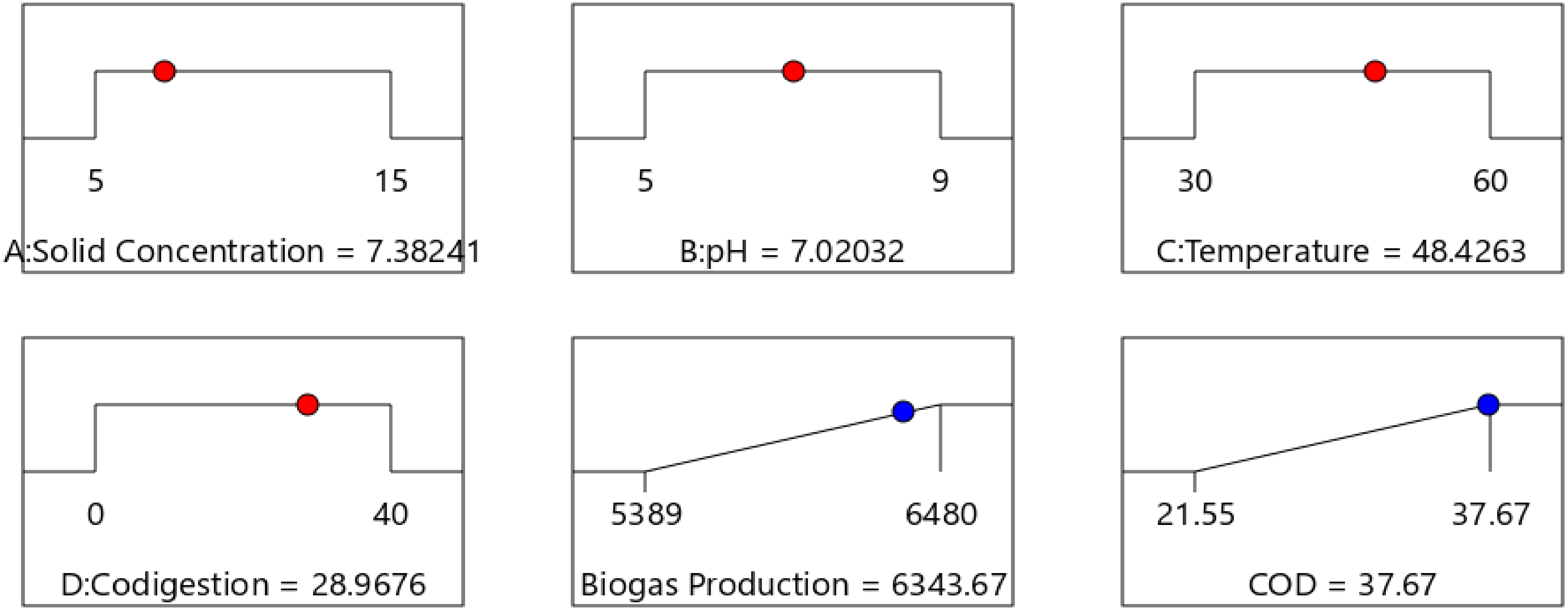
VARIOUS PLOTS IN RSM

8. 3D Surface plot - Surface plots are diagrams of 3D data. Rather than showing the individual data points, surface plots show a functional relationship between a designated dependent variable (Y), and two independent variables (X and Z). The plot is a companion plot to the contour plot.
9. Cube plot - The cube plot shows the predicted values from the coded model for the combinations of the -1 and +1 levels of any three factors.

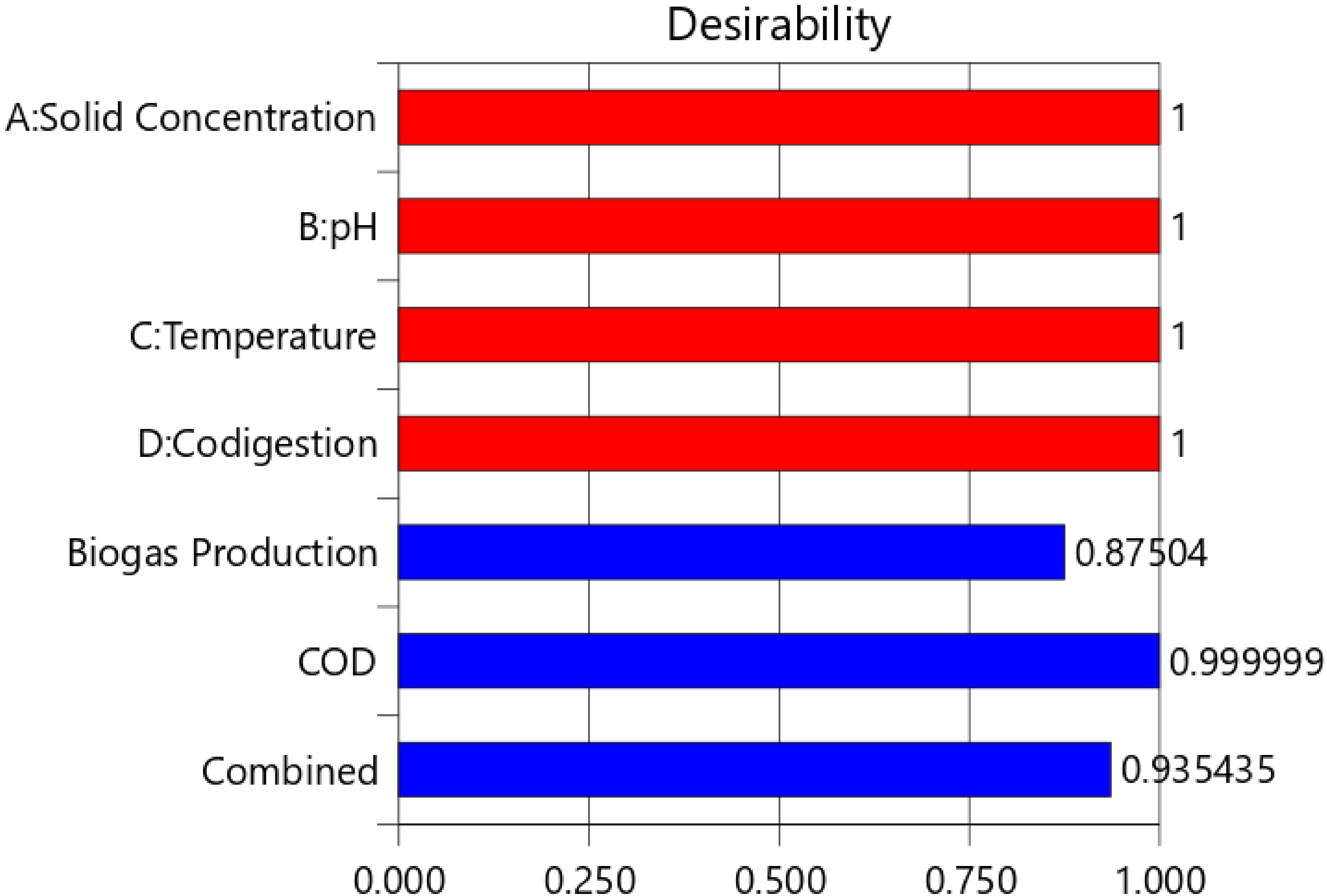

Cube
COD
32.6441
31.2513
D+: 40
32.6516
32.9513
D: Codigestion
25.188
27.9327
A+: 15
A: Solid Concentration
D-: 0
24.908
29.3452
A-: 5
B-: 5
B: pH
B+: 9

VARIOUS PLOTS IN RSM

10. Ramp plot- The ramp display combines individual graphs for easier interpretation. The coloured dot on each ramp reflects the factor setting or response prediction for that solution. The height of the dot shows how desirable it is.
11. Desirability plot - The bar graph shows how well each variable satisfies the criteria: values near one are good.



Desirability = 0.935
Solution 1 out of 18


Desirability
A:Solid Concentration
B:pH
C:Temperature
D:Codigestion
Biogas Production
COD
Combined
1
1
1
1
0.87504
0.999999
0.935435
0.000
0.250
0.500
0.750
1.000

THANK YOU